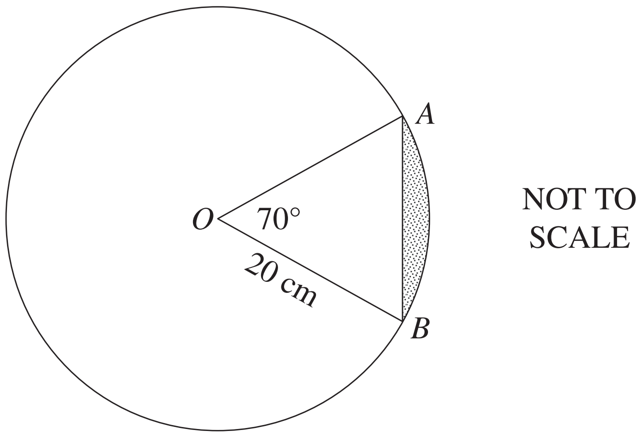
Question 13 (15 marks) Use the Question 13 Writing Booklet.

(a) Solve $2\sin x\cos x = \sin x$ for $0 \le x \le 2\pi$. **3**

(b) The diagram shows a circle with centre O and radius 20 cm. **3**

The points A and B lie on the circle such that $\angle AOB = 70°$.



Find the perimeter of the shaded segment, giving your answer correct to one decimal place.

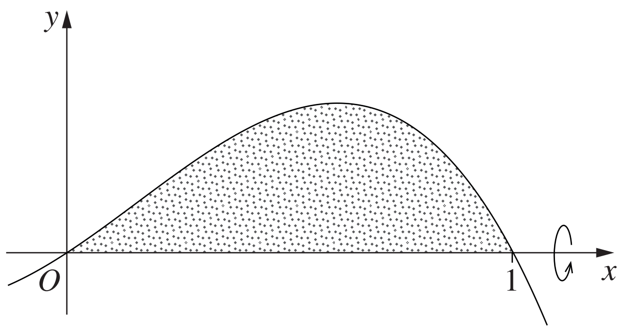
(c) (i) Differentiate $(\ln x)^2$. **2**

(ii) Hence, or otherwise, find **1**

$$\int \frac{\ln x}{x}\,dx.$$

Question 13 continues on page 11

(d) The diagram shows the region bounded by the curve $y = x - x^3$, and the x-axis between $x = 0$ and $x = 1$. The region is rotated about the x-axis to form a solid. **3**



Find the exact value of the volume of the solid formed.

(e) (i) Sketch the graph of $y = |x - 1|$ for $-4 \le x \le 4$. **1**

(ii) Using the sketch from part (i), or otherwise, solve $|x - 1| = 2x + 4$. **2**

End of Question 13

Question 14 (15 marks) Use the Question 14 Writing Booklet.

(a) A particle is moving along a straight line. The particle is initially at rest. The acceleration of the particle at time t seconds is given by $a = e^{2t} - 4$, where $t \geq 0$. **2**

Find an expression, in terms of t, for the velocity of the particle.

(b) The derivative of a function $y = f(x)$ is given by $f'(x) = 3x^2 + 2x - 1$.

(i) Find the x-values of the two stationary points of $y = f(x)$, and determine the nature of the stationary points. **2**

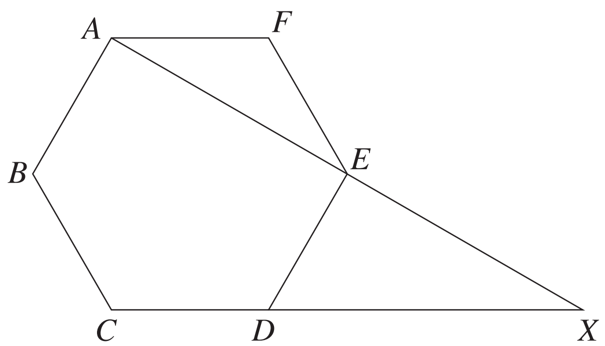
(ii) The curve passes through the point (0, 4). **2**

Find an expression for $f(x)$.

(iii) Hence sketch the curve, clearly indicating the stationary points. **2**

(iv) For what values of x is the curve concave down? **1**

(c) The regular hexagon $ABCDEF$ has sides of length 1. The diagonal AE and the side CD are produced to meet at the point X. **3**



Copy or trace the diagram into your writing booklet.

Find the exact length of the line segment EX, justifying your answer.

(d) The equation of the tangent to the curve $y = x^3 + ax^2 + bx + 4$ at the point where $x = 2$ is $y = x - 4$. **3**

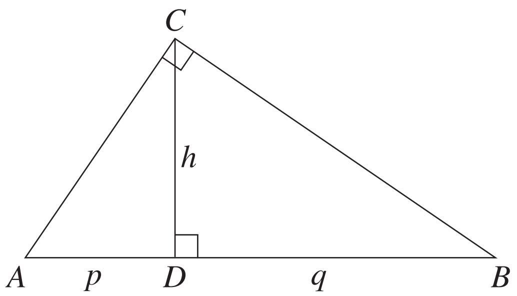
Find the values of a and b.

Question 15 (15 marks) Use the Question 15 Writing Booklet.

(a) Solve $e^{2\ln x} = x + 6$. **2**

(b) The triangle ABC is a right-angled triangle with the right angle at C. The point D is chosen on AB so that CD is perpendicular to AB. **3**

The length of AD is p, the length of BD is q and the length of CD is h.



Show that $h = \sqrt{pq}$.

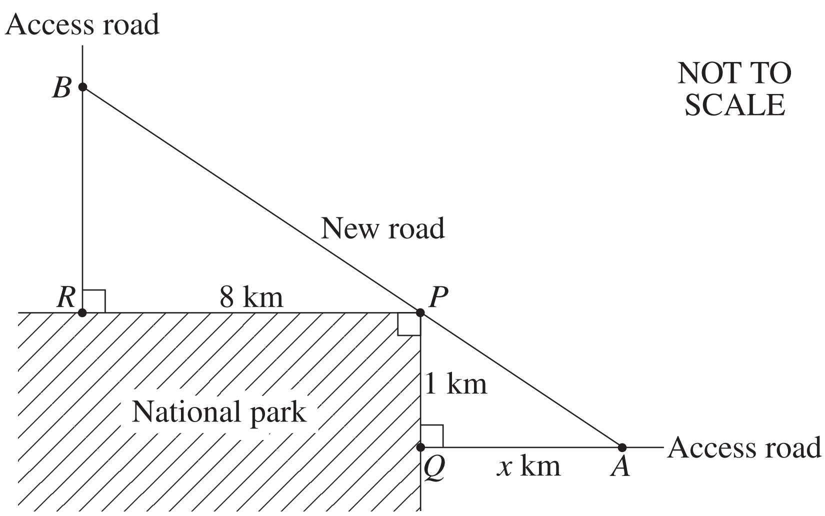
Question 15 continues on page 14

(c) The entry points, R and Q, to a national park can be reached via two straight access roads. The access roads meet the national park boundaries at right angles. The corner, P, of the national park is 8 km from R and 1 km from Q. The boundaries of the national park form a right angle at P.

A new straight road is to be built joining these roads and passing through P.

Points A and B on the access roads are to be chosen to minimise the distance, D km, from A to B along the new road.

Let the distance QA be x km.



(i) Show that $D^2 = (x + 8)^2 + \left(\frac{8}{x} + 1\right)^2$. **3**

(ii) Show that $x = 2$ gives the minimum value of D^2. **3**

(d) The probability that a person chosen at random has red hair is 0.02.

(i) Two people are chosen at random. **2**

What is the probability that at least ONE has red hair?

(ii) What is the smallest number of people that can be chosen at random so that the probability that at least ONE has red hair is greater than 0.4? **2**

End of Question 15

Question 16 (15 marks) Use the Question 16 Writing Booklet.

(a) A person wins \$1 000 000 in a competition and decides to invest this money in an account that earns interest at 6% per annum compounded quarterly. The person decides to withdraw \$80 000 from this account at the end of every fourth quarter. Let A_n be the amount remaining in the account after the nth withdrawal.

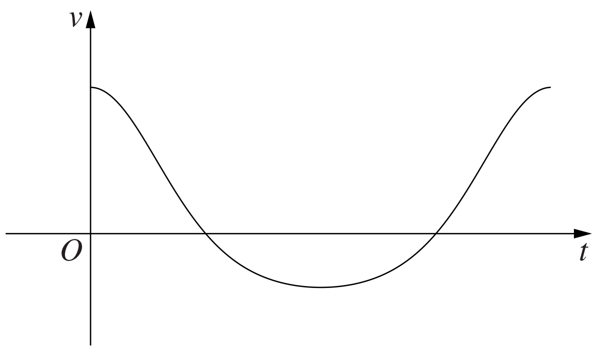
(i) Show that the amount remaining in the account after the withdrawal at the end of the eighth quarter is **2**

$$A_2 = 1\,000\,000 \times 1.015^8 - 80\,000\left(1 + 1.015^4\right).$$

(ii) For how many years can the full amount of \$80 000 be withdrawn? **3**

(b) A particle moves in a straight line, starting at the origin. Its velocity, v m s^{-1}, is given by $v = e^{\cos t} - 1$, where t is in seconds. **3**

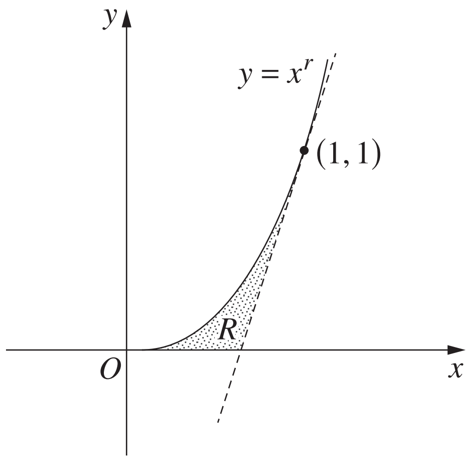
The diagram shows the graph of the velocity against time.



Using one application of Simpson's rule, estimate the position of the particle when it first comes to rest. Give your answer correct to two decimal places.

Question 16 continues on page 16

(c) The diagram shows the region R, bounded by the curve $y = x^r$, where $r \geq 1$, the x-axis and the tangent to the curve at the point $(1, 1)$.



(i) Show that the tangent to the curve at $(1, 1)$ meets the x-axis at $\left(\dfrac{r-1}{r}, 0\right)$. **2**

(ii) Using the result of part (i), or otherwise, show that the area of the region R is $\dfrac{r-1}{2r(r+1)}$. **2**

(iii) Find the exact value of r for which the area of R is a maximum. **3**

End of paper

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2019 HIGHER SCHOOL CERTIFICATE EXAMINATION

REFERENCE SHEET

– Mathematics –

– Mathematics Extension 1 –

– Mathematics Extension 2 –

1271

Mathematics

Factorisation

$$a^2 - b^2 = (a+b)(a-b)$$

$$a^3 + b^3 = (a+b)\left(a^2 - ab + b^2\right)$$

$$a^3 - b^3 = (a-b)\left(a^2 + ab + b^2\right)$$

Angle sum of a polygon

$$S = (n-2)\times 180^\circ$$

Equation of a circle

$$(x-h)^2 + (y-k)^2 = r^2$$

Trigonometric ratios and identities

$$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

$$\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}}$$

$$\operatorname{cosec}\theta = \frac{1}{\sin\theta}$$

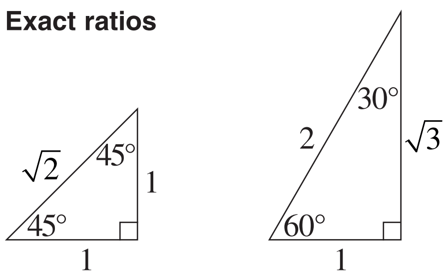
$$\sec\theta = \frac{1}{\cos\theta}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\sin^2\theta + \cos^2\theta = 1$$

Exact ratios



Sine rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Cosine rule

$$c^2 = a^2 + b^2 - 2ab\cos C$$

Area of a triangle

$$\text{Area} = \frac{1}{2}ab\sin C$$

Distance between two points

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Perpendicular distance of a point from a line

$$d = \frac{\left|ax_1 + by_1 + c\right|}{\sqrt{a^2 + b^2}}$$

Slope (gradient) of a line

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Point–gradient form of the equation of a line

$$y - y_1 = m(x - x_1)$$

***n*th term of an arithmetic series**

$$T_n = a + (n-1)d$$

Sum to *n* terms of an arithmetic series

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right] \quad \text{or} \quad S_n = \frac{n}{2}(a+l)$$

***n*th term of a geometric series**

$$T_n = ar^{n-1}$$

Sum to *n* terms of a geometric series

$$S_n = \frac{a\left(r^n - 1\right)}{r-1} \quad \text{or} \quad S_n = \frac{a\left(1 - r^n\right)}{1-r}$$

Limiting sum of a geometric series

$$S = \frac{a}{1-r}$$

Compound interest

$$A_n = P\left(1 + \frac{r}{100}\right)^n$$

Differentiation from first principles

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Derivatives

If $y = x^n$, then $\dfrac{dy}{dx} = nx^{n-1}$

If $y = uv$, then $\dfrac{dy}{dx} = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$

If $y = \dfrac{u}{v}$, then $\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

If $y = F(u)$, then $\dfrac{dy}{dx} = F'(u)\dfrac{du}{dx}$

If $y = e^{f(x)}$, then $\dfrac{dy}{dx} = f'(x)e^{f(x)}$

If $y = \log_e f(x) = \ln f(x)$, then $\dfrac{dy}{dx} = \dfrac{f'(x)}{f(x)}$

If $y = \sin f(x)$, then $\dfrac{dy}{dx} = f'(x)\cos f(x)$

If $y = \cos f(x)$, then $\dfrac{dy}{dx} = -f'(x)\sin f(x)$

If $y = \tan f(x)$, then $\dfrac{dy}{dx} = f'(x)\sec^2 f(x)$

Solution of a quadratic equation

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Sum and product of roots of a quadratic equation

$\alpha + \beta = -\dfrac{b}{a}$ $\qquad$ $\alpha\beta = \dfrac{c}{a}$

Equation of a parabola

$$(x-h)^2 = \pm 4a(y-k)$$

Integrals

$$\int (ax+b)^n \, dx = \frac{(ax+b)^{n+1}}{a(n+1)} + C$$

$$\int e^{ax+b} \, dx = \frac{1}{a}e^{ax+b} + C$$

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + C$$

$$\int \sin(ax+b)\,dx = -\frac{1}{a}\cos(ax+b) + C$$

$$\int \cos(ax+b)\,dx = \frac{1}{a}\sin(ax+b) + C$$

$$\int \sec^2(ax+b)\,dx = \frac{1}{a}\tan(ax+b) + C$$

Trapezoidal rule (one application)

$$\int_a^b f(x)\,dx \approx \frac{b-a}{2}\left[f(a) + f(b)\right]$$

Simpson's rule (one application)

$$\int_a^b f(x)\,dx \approx \frac{b-a}{6}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right]$$

Logarithms – change of base

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Angle measure

$180° = \pi$ radians

Length of an arc

$l = r\theta$

Area of a sector

$\text{Area} = \dfrac{1}{2}r^2\theta$

Mathematics Extension 1

Angle sum identities

$$\sin(\theta + \phi) = \sin\theta\cos\phi + \cos\theta\sin\phi$$

$$\cos(\theta + \phi) = \cos\theta\cos\phi - \sin\theta\sin\phi$$

$$\tan(\theta + \phi) = \frac{\tan\theta + \tan\phi}{1 - \tan\theta\tan\phi}$$

***t* formulae**

If $t = \tan\frac{\theta}{2}$, then

$$\sin\theta = \frac{2t}{1+t^2}$$

$$\cos\theta = \frac{1-t^2}{1+t^2}$$

$$\tan\theta = \frac{2t}{1-t^2}$$

General solution of trigonometric equations

$\sin\theta = a, \qquad \theta = n\pi + (-1)^n \sin^{-1} a$

$\cos\theta = a, \qquad \theta = 2n\pi \pm \cos^{-1} a$

$\tan\theta = a, \qquad \theta = n\pi + \tan^{-1} a$

Division of an interval in a given ratio

$$\left(\frac{mx_2 + nx_1}{m+n}, \ \frac{my_2 + ny_1}{m+n}\right)$$

Parametric representation of a parabola

For $x^2 = 4ay$,

$$x = 2at, \quad y = at^2$$

At $\left(2at, at^2\right)$,

tangent: $y = tx - at^2$

normal: $x + ty = at^3 + 2at$

At $\left(x_1, y_1\right)$,

tangent: $xx_1 = 2a\left(y + y_1\right)$

normal: $y - y_1 = -\frac{2a}{x_1}\left(x - x_1\right)$

Chord of contact from $\left(x_0, y_0\right)$: $xx_0 = 2a\left(y + y_0\right)$

Acceleration

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

Simple harmonic motion

$$x = b + a\cos\left(nt + \alpha\right)$$

$$\ddot{x} = -n^2\left(x - b\right)$$

Further integrals

$$\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \sin^{-1}\frac{x}{a} + C$$

$$\int \frac{1}{a^2 + x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a} + C$$

Sum and product of roots of a cubic equation

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

$$\alpha\beta\gamma = -\frac{d}{a}$$

Estimation of roots of a polynomial equation

Newton's method

$$x_2 = x_1 - \frac{f\left(x_1\right)}{f'\left(x_1\right)}$$

Binomial theorem

$$\left(a + b\right)^n = \sum_{k=0}^{n}\binom{n}{k}a^k b^{n-k} = \sum_{k=0}^{n}\binom{n}{k}a^{n-k}b^k$$